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Spatial-Color Pixel Classification by Spectral Clustering for Color Image Segmentation

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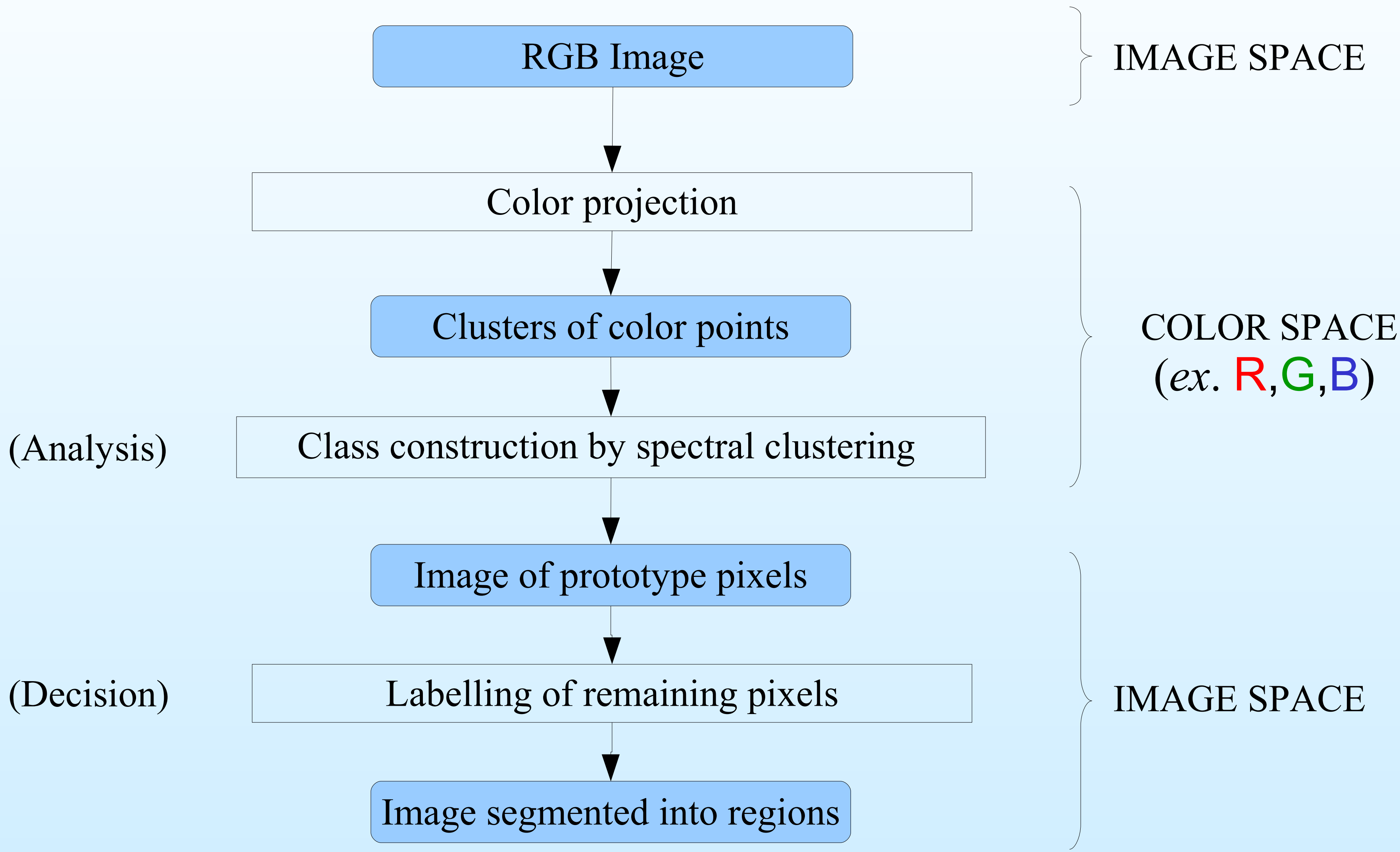
Summary

- **Context**
  - Segmentation by pixel classification
  - Similarity between pixels
- Similarity matrix between colors
  - Spatial-color compactness degree
  - Similarity matrix
- Spectral clustering
  - A clustering method based upon the spectral decomposition of a similarity matrix
  - Algorithms
- Experimental results
- Conclusions and future work

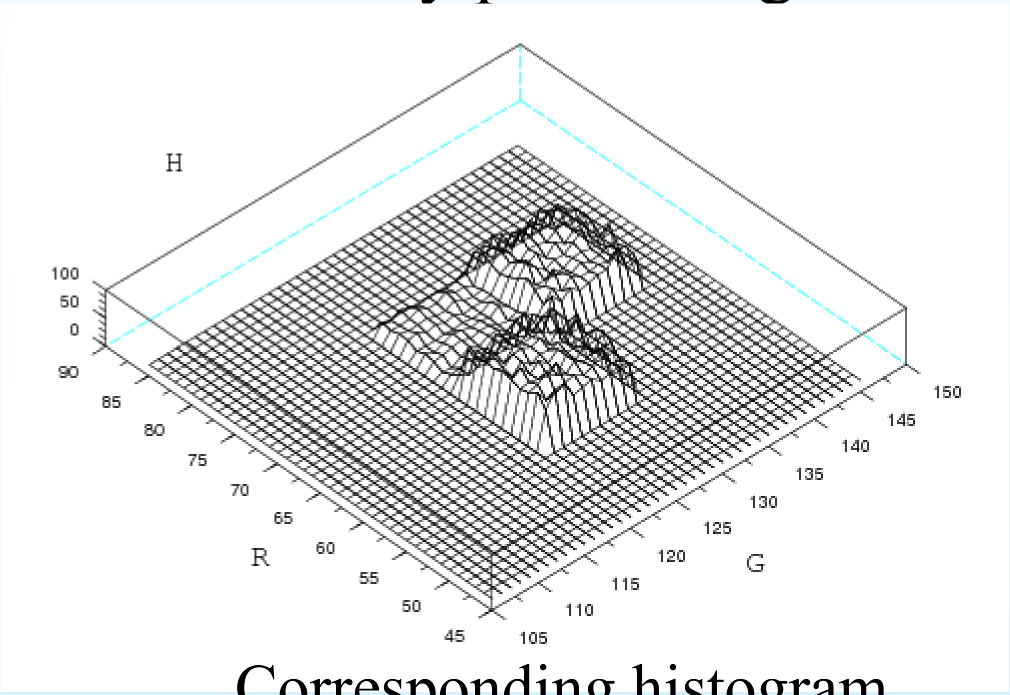
Segmentation by pixel classification



Histogram limitations

- Histogram shortcomings for segmentation
  - Overlapping distributions ⇨ tricky processing

Synthetic image



Corresponding histogram

  - No information about the location of colors within the image

↳ integrate homogeneity ***and*** connectedness

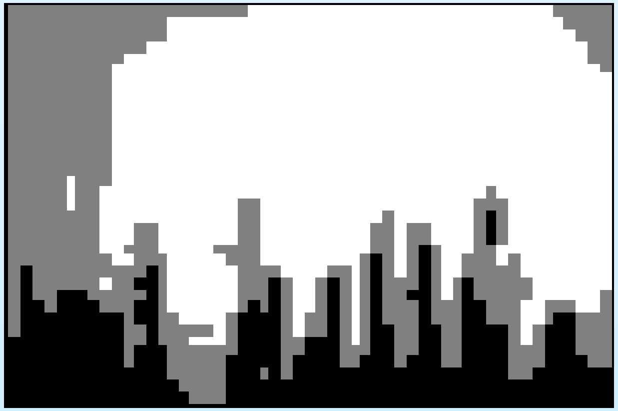
Similarity between pixels(1)

- Colors of pixels P,Q: RGB(P),RGB(Q)
- Spatial locations of P,Q: XY(P),XY(Q)
- How combine spatial locations and colors?
- Classical technique:
- $\Phi(P,Q) = \alpha.e^{-(RGB(P)-RGB(Q))^2}.\beta.e^{-(XY(P)-XY(Q))^2}$
- Very high size of the matrix
- Solution: square blocs of pixels

Similarity between pixels(2)

Image

Spectral clustering



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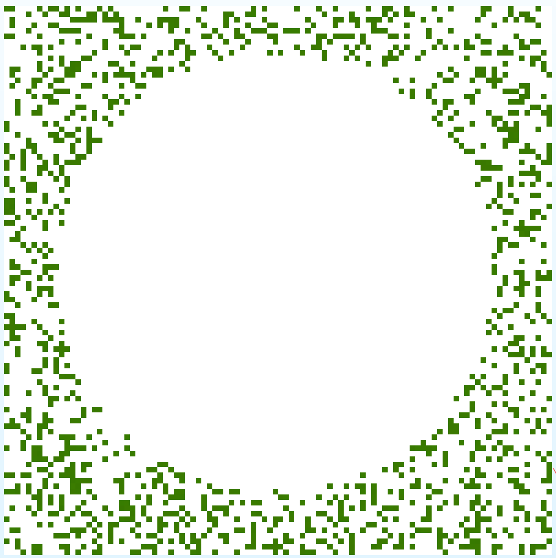
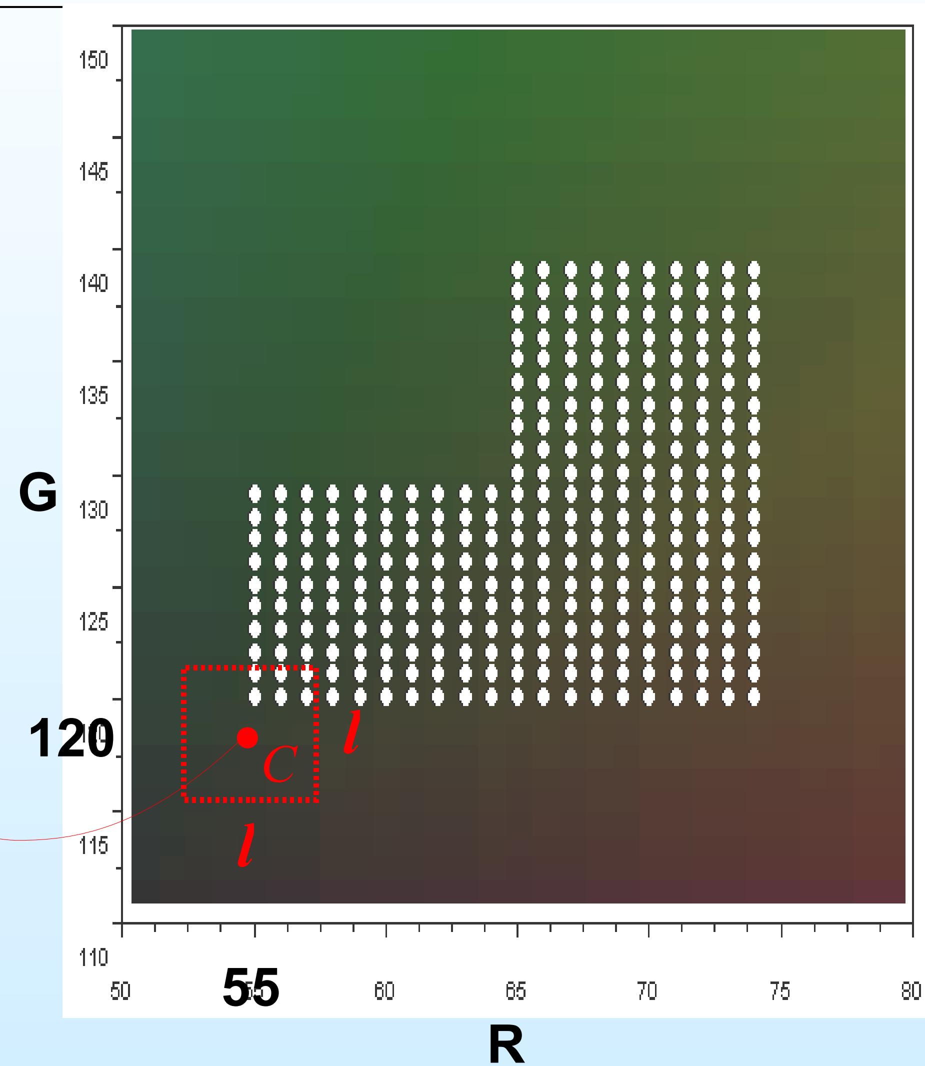
Spatial color compactness degree (1)



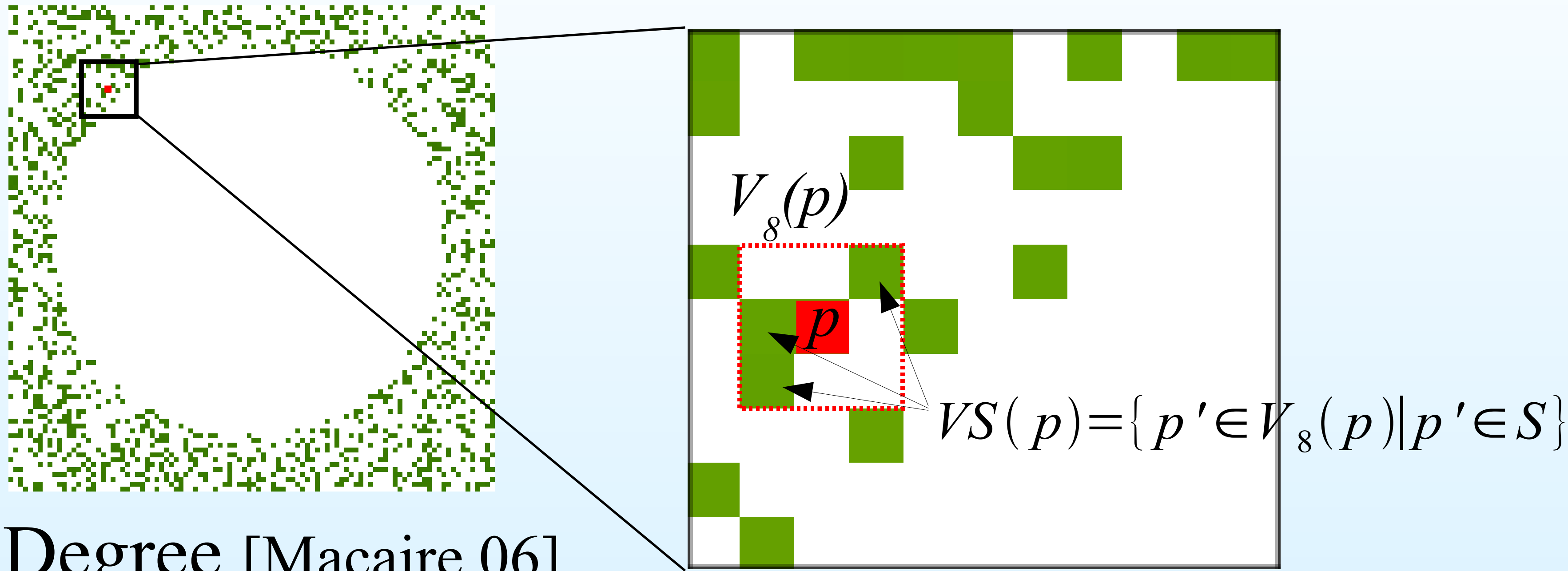


Color space C

- Definitions
  - Color point

    $C=[c^R,c^G,c^B]^T$
  - Color domain $\boldsymbol{D}_l(C)$
  - Pixel subset $S_l(C)$

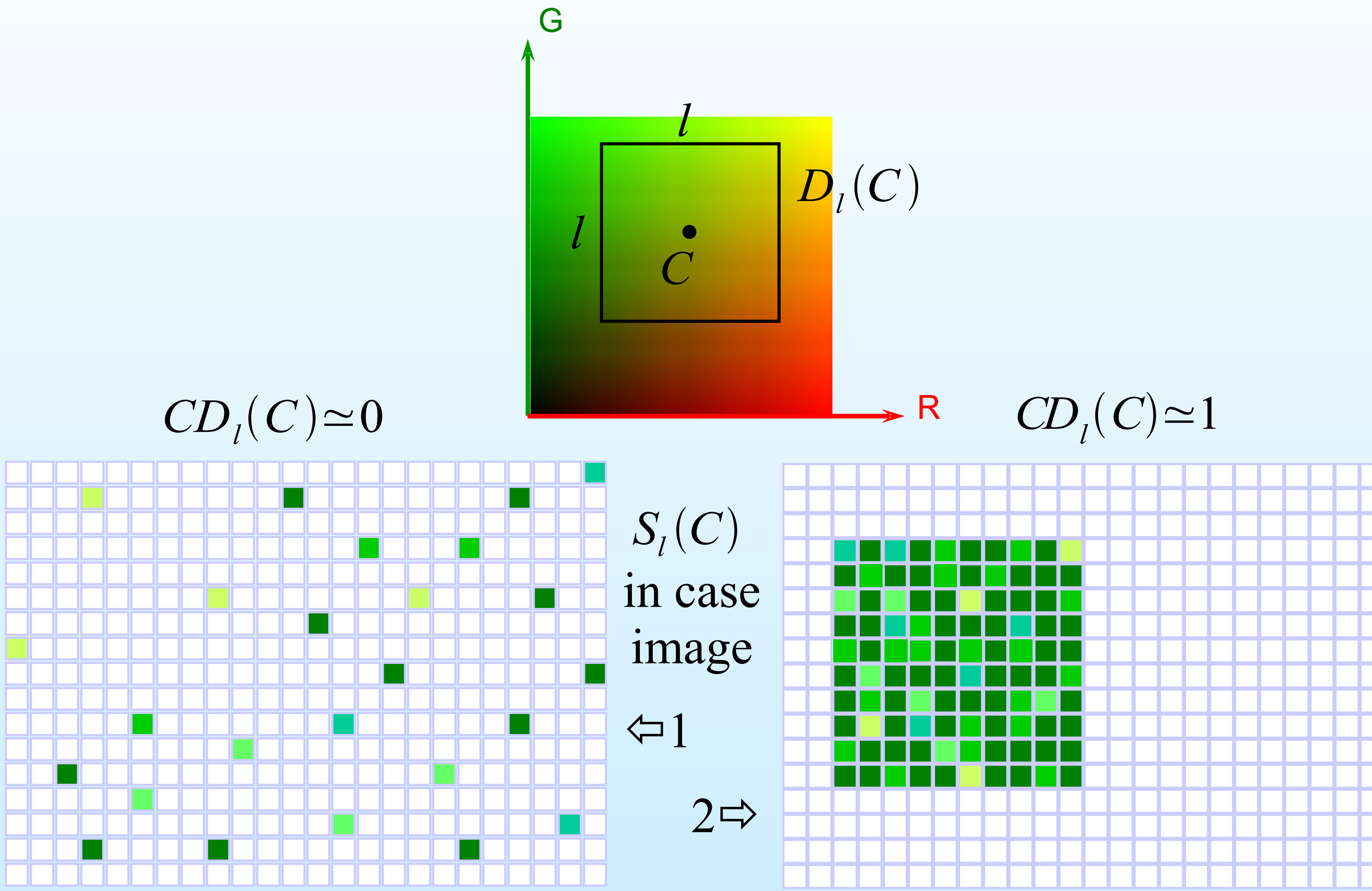
Spatial color compactness degree (2)



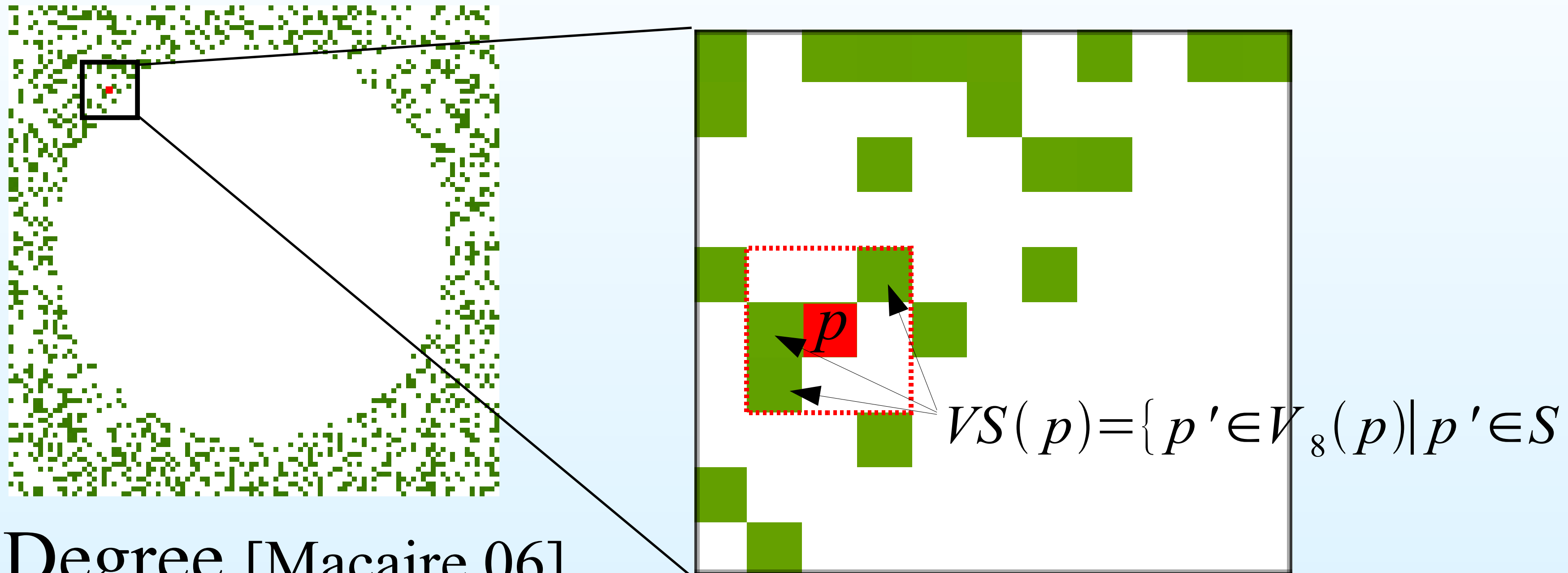
- Degree [Macaire 06]
  - of spatial connectedness

$$CD_l(C)=\frac{1}{|S_l(C)|}\sum_{p\in S_l(C)}\frac{|VS(p)|}{8}$$

Spatial color compactness degree (2)



Spatial color compactness degree (3)

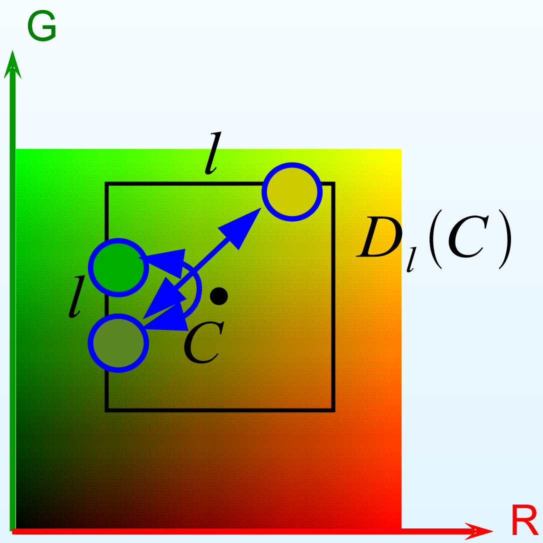


- Degree [Macaire 06]
  - of color homogeneity

$$HD_l(C) = \frac{\textit{average local}\text{ color }\textit{dispersion of } S_l(C)}{\textit{global}\text{ color }\textit{dispersion of } S_l(C)}$$

$$= \frac{\overline{\sigma_{VS}}}{\sigma_S} = \frac{\frac{1}{|S_l(C)|}\sum_{p \in S_l(C)} \sigma_{VS}(p)}{\sigma_S}$$

Spatial color compactness degree (3)

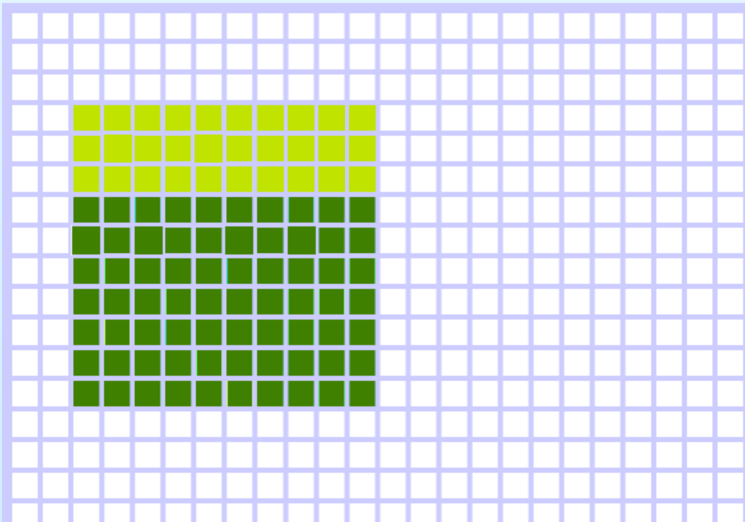


$\overline{\sigma_{VS}} \ll \sigma_S$

$\rightarrow HD_l(C) \simeq 0$

$\overline{\sigma_{VS}} \simeq \sigma_S$

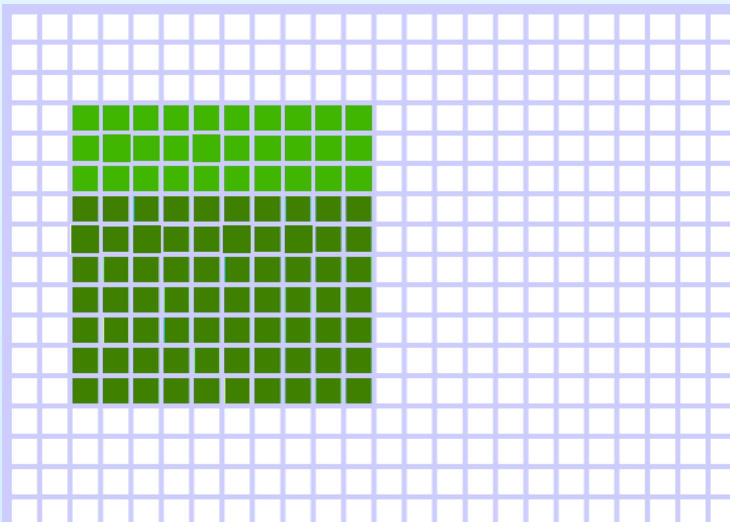
$\rightarrow HD_l(C) \simeq 1$



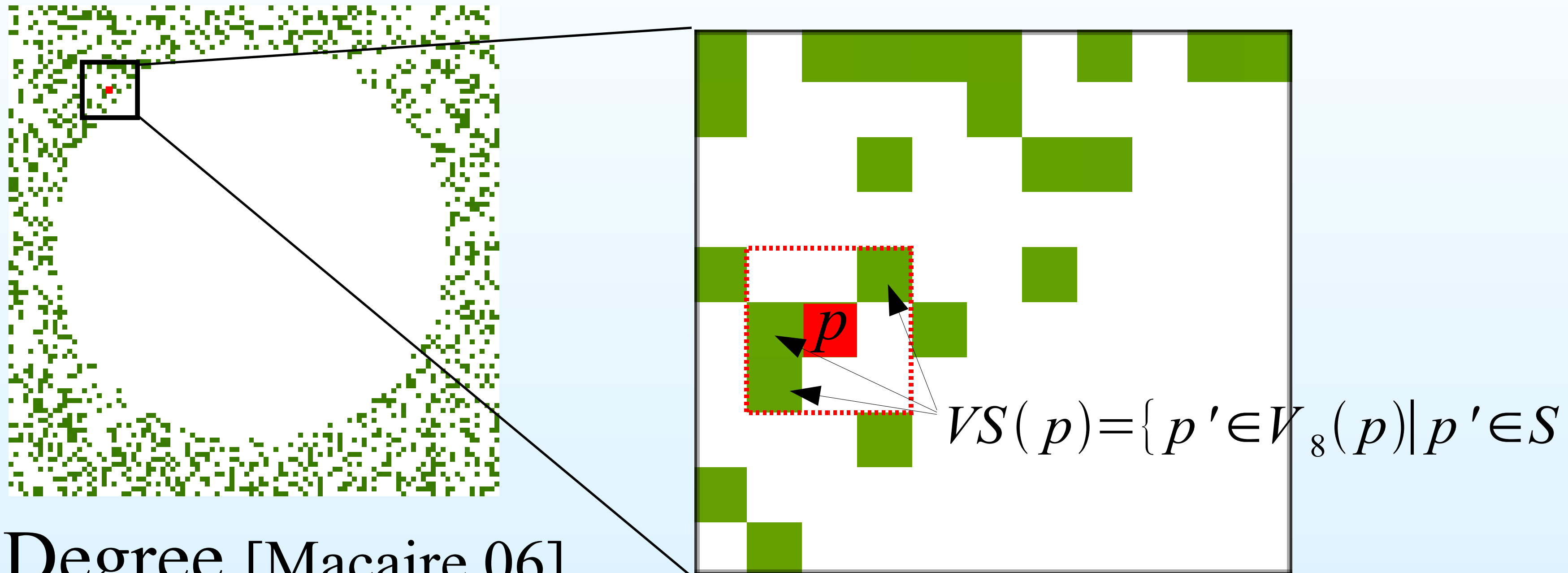
$S_l(C)$ in case image

⇦1

2⇨



Spatial color compactness degree (4)



- Degree [Macaire 06]
  - of spatio-color compactness

$$SCCD_l(C)=CD_l(C)\times HD_l(C)$$

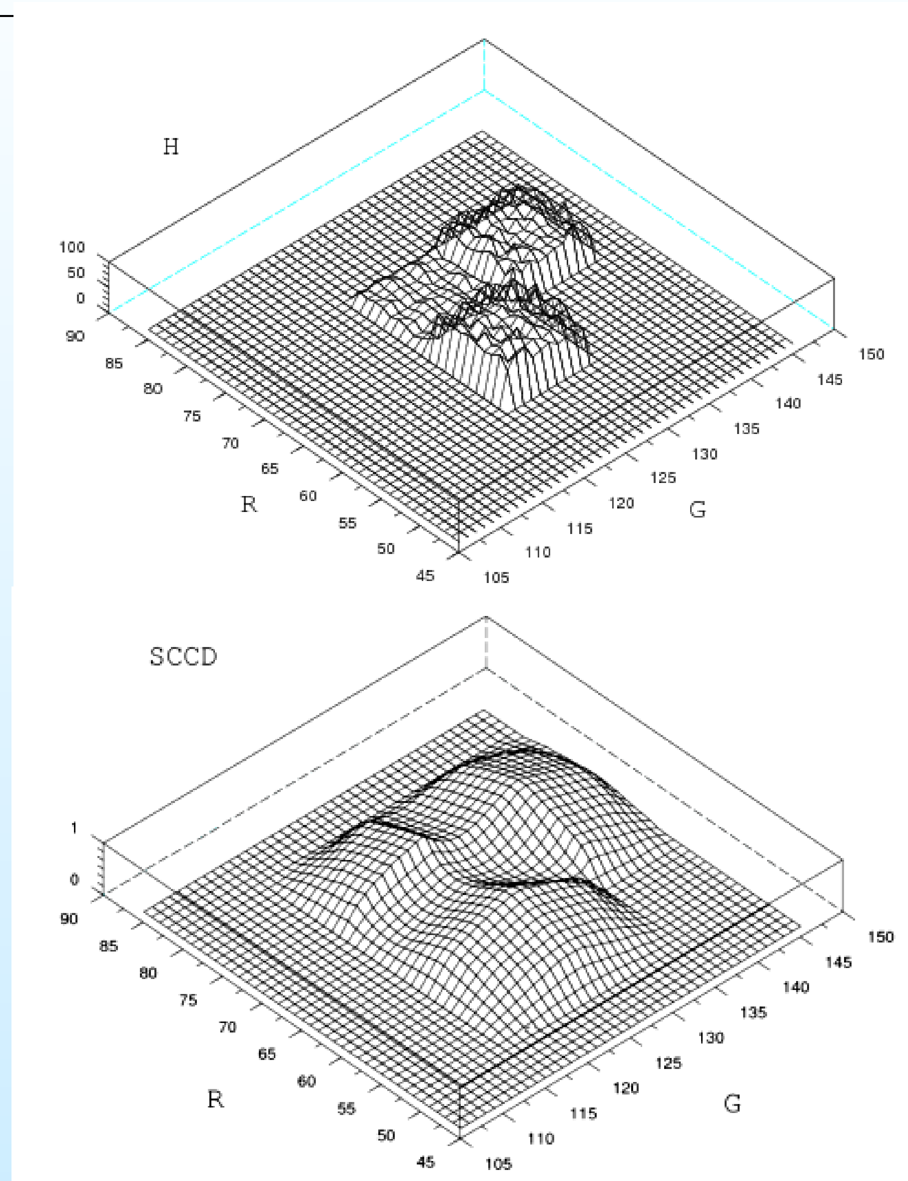
$SCCD_l(C)\simeq 1^-$ ⇔ pixels in $S_l(C)$ highly connected and their color points concentrated in C

$SCCD_l(C)\simeq 0^+$ ⇔ pixels in $S_l(C)$ scattered or/and their color points dispersed in C

Spatial color compactness degree (5)

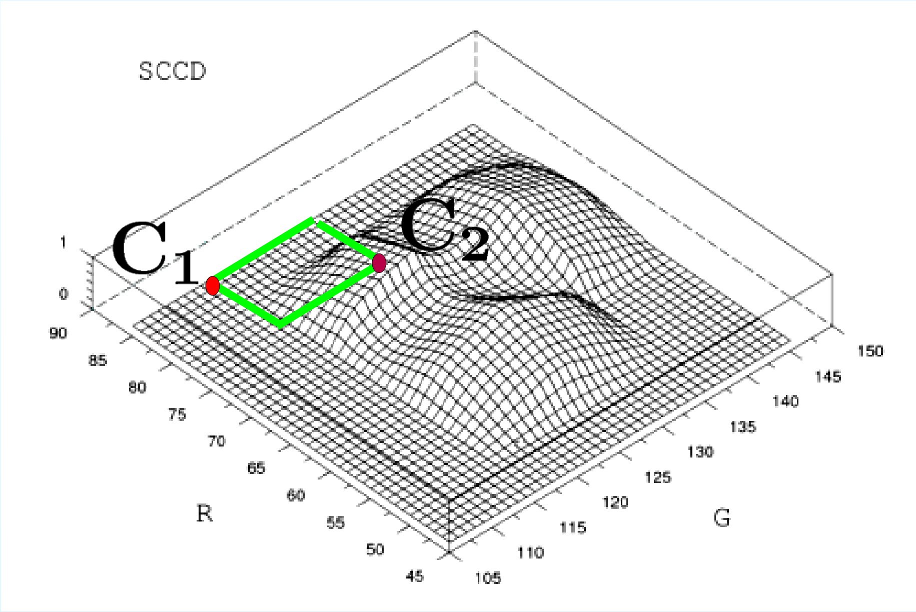
Histogram

SCCD



Similarity matrix (1)

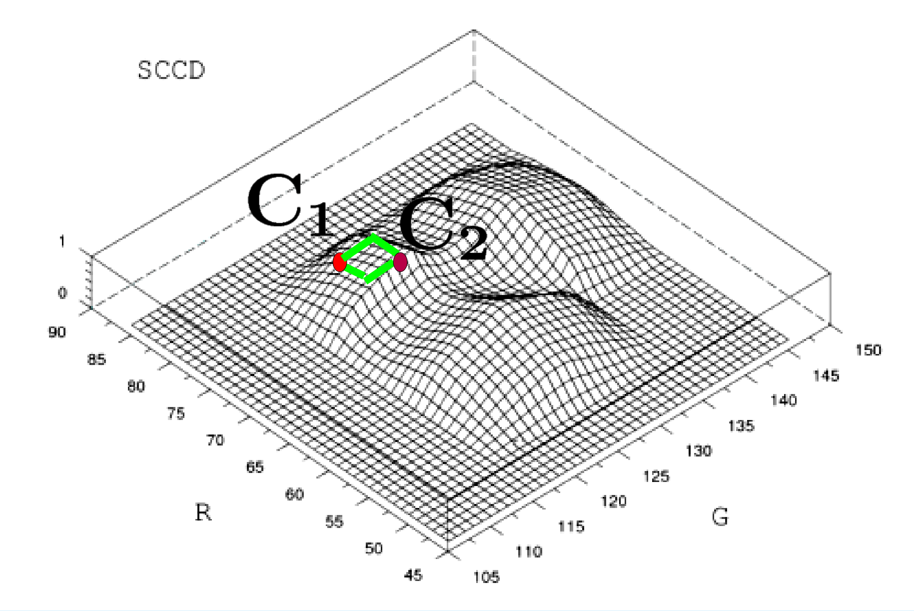
$$\Phi(\mathbf{C_1}, \mathbf{C_2}) = \min_{\mathbf{C} \in \underline{D}(\mathbf{C_1}, \mathbf{C_2})} SCCD(\mathbf{C}, l)$$



$\Phi(\mathbf{C_1}, \mathbf{C_2})$ close to 0

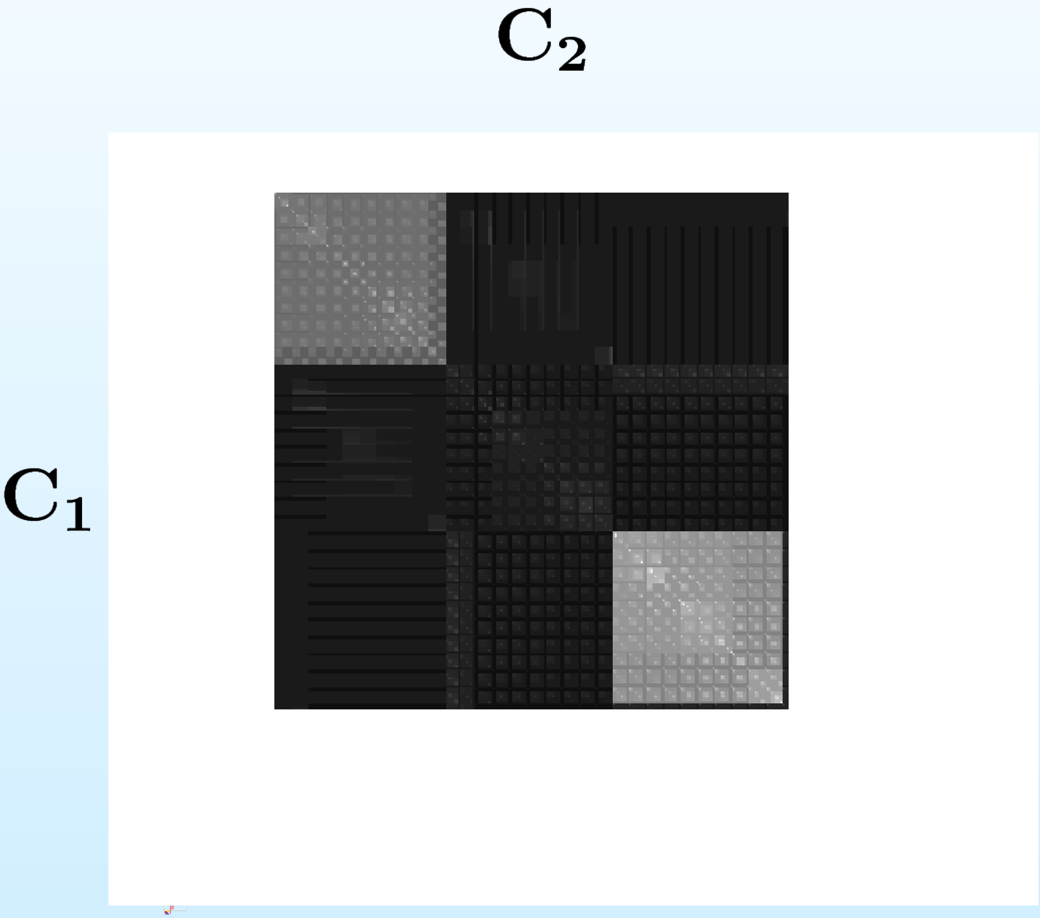
Similarity matrix (2)

$$\Phi(\mathbf{C_1}, \mathbf{C_2}) = \min_{\mathbf{C} \in \underline{D}(\mathbf{C_1}, \mathbf{C_2})} SCCD(\mathbf{C}, l)$$



$\Phi(\mathbf{C_1}, \mathbf{C_2})$ close to 1

Similarity matrix (3)



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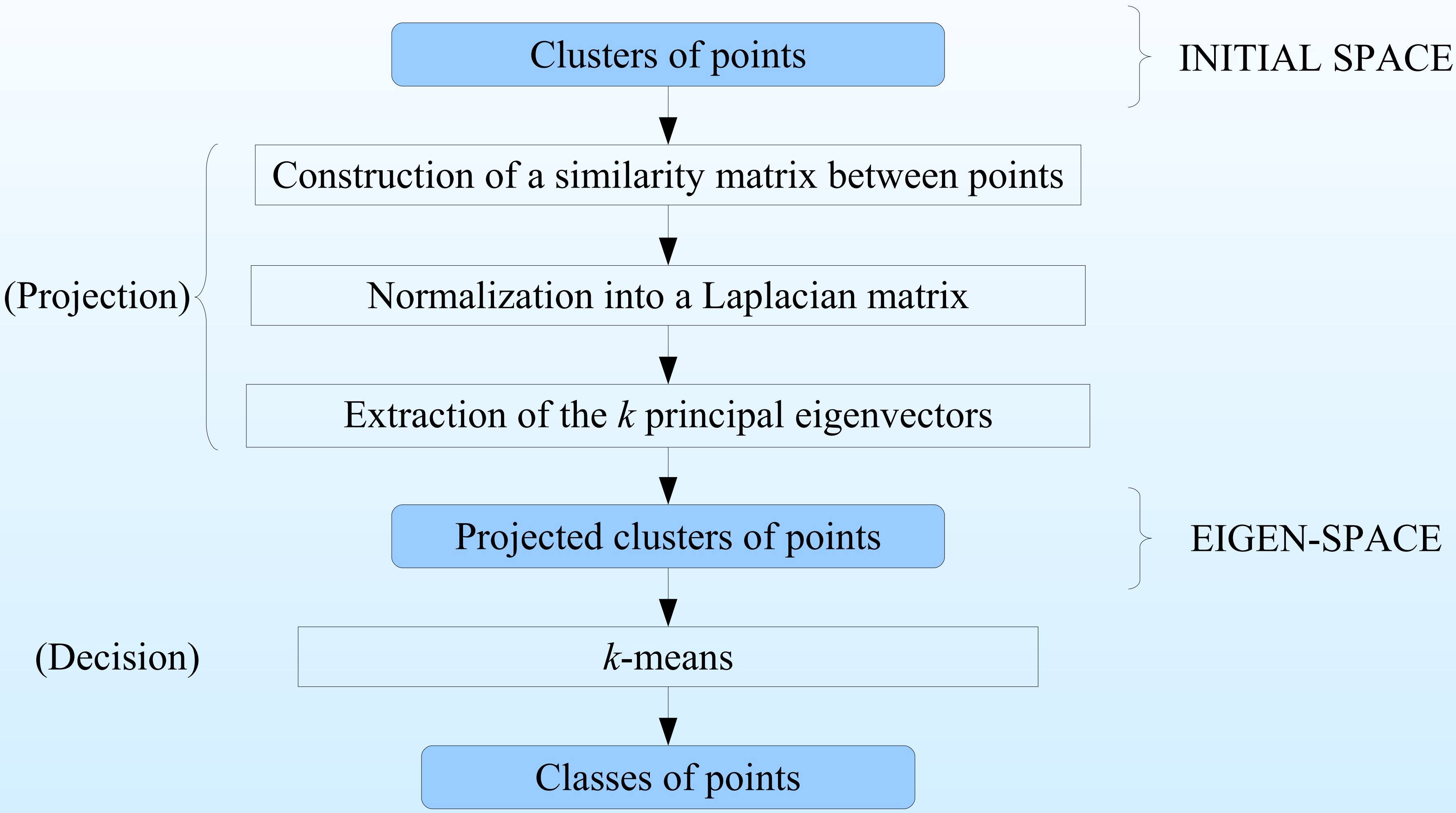
Spectral clustering overview (1)

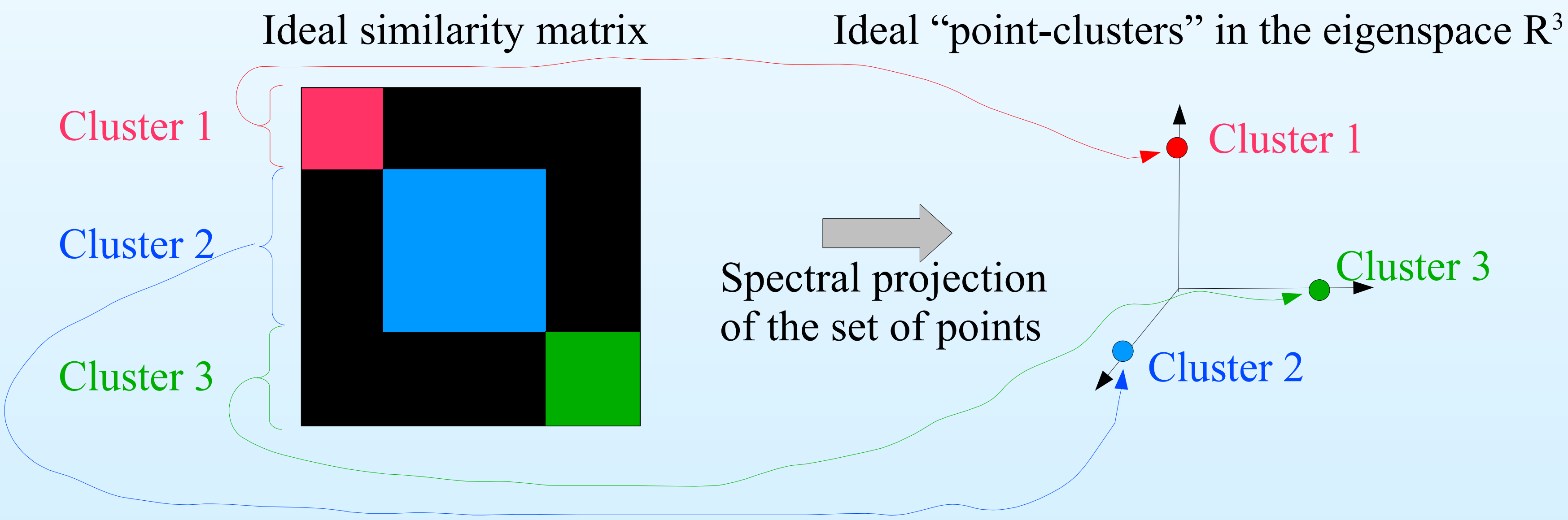
- Clustering based upon the spectral decomposition of a similarity matrix.
- Derived from the graph theory with the minimized cuts problems.
- Main advantages:
  - Ability to recognize non-convex clusters.
  - Non-constrained measurement of the similarity matrix between points.
  - Optimization procedure without any local minima.

Spectral clustering overview (2)



Spectral clustering justifications (1)

- Ng's Algorithm
  - Ideal case when a permutation of points induces a block-diagonal similarity matrix.

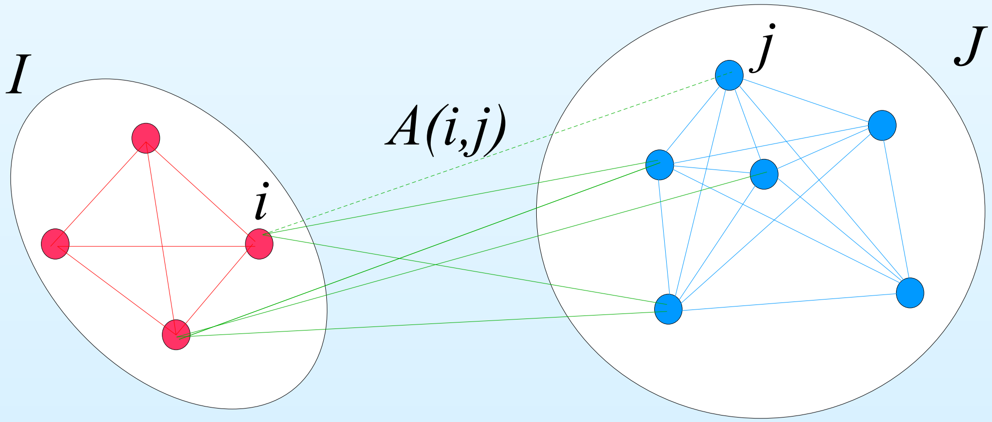


  - Non-ideal case: proof of a good robustness to some perturbations of the matrix similarity.

Spectral clustering justifications (2)

- Shi's algorithm
  - Recursive bi-classes or *k*-classes clustering.
  - Eigenvectors solve a relaxed version of the *normalized-cut* criterion.

k=2



$$Cut(I,J)=\sum_{i\in I,\, j\in J} A(i,j)$$

$$Assoc(I)=\sum_{x\in I,\, y\in I} A(x,y)$$

$$NCut(I,J)=\frac{Cut(I,J)}{Assoc(I,I\cup J)}+\frac{Cut(I,J)}{Assoc(J,I\cup J)}$$

Ng's algorithm

- Build a similarity matrix A: $A(i,j) \in [0,1]$
- Process the diagonal matrix D by: $D(i,i) = \sum_j A(i,j)$
- Define the Laplacian matrix L: $L = D^{-1/2} A D^{-1/2}$
- Extract the k principal eigenvectors of L: $\{X_1, \ldots, X_k\}$
- Compute the matrix Y by normalizing each line of X:

$$Y_{ij} = X_{ij} / \sum_m X_{im}^2$$

- Apply a k-means on the set of points C_i whose coordinates are: $Y(i,m), m \in \{1, \ldots, k\}$

Shi's algorithm

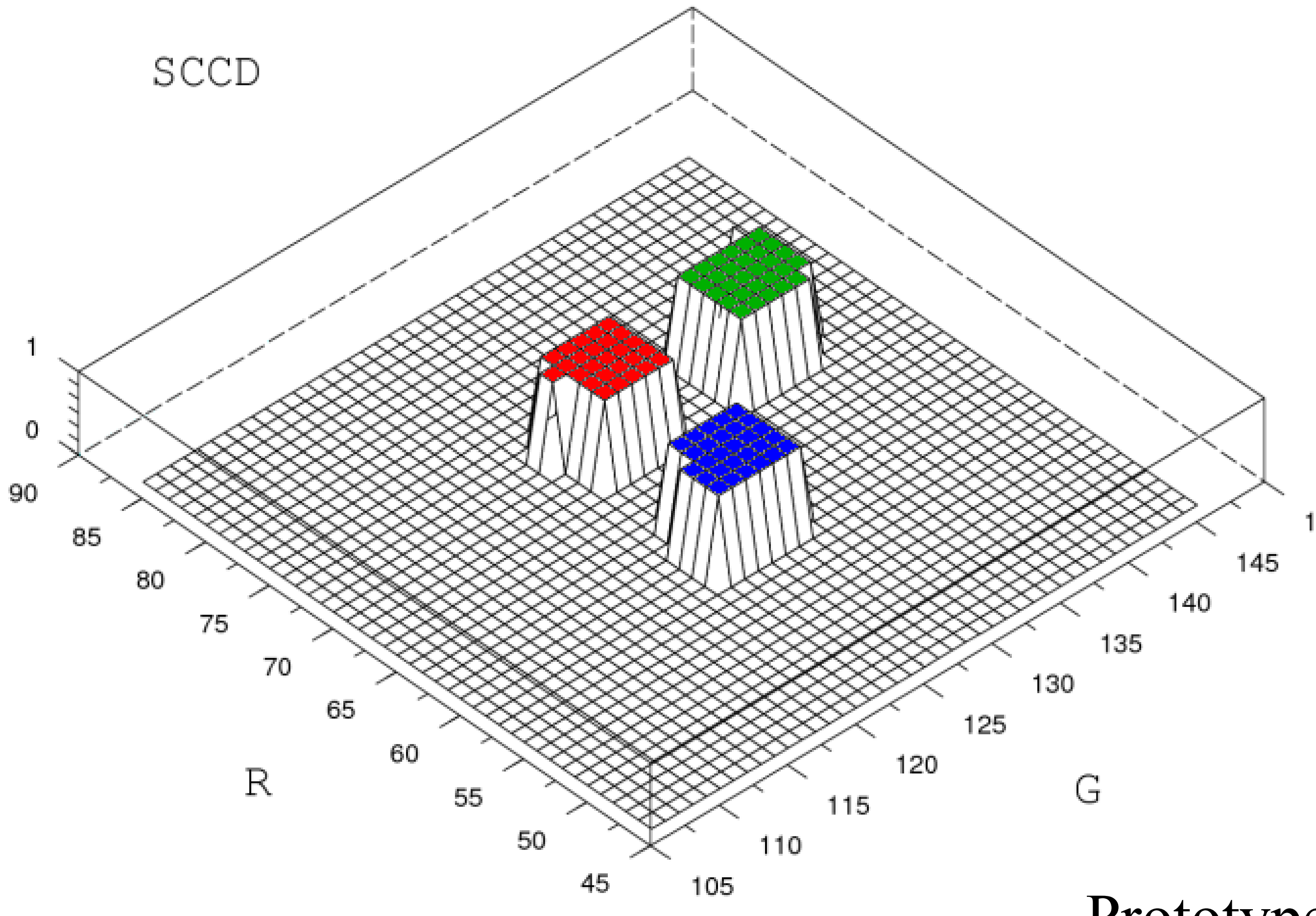
- Non-recursive version: "*k-way cut*" clustering.
- Eigenvectors are not normalized, but weighted:

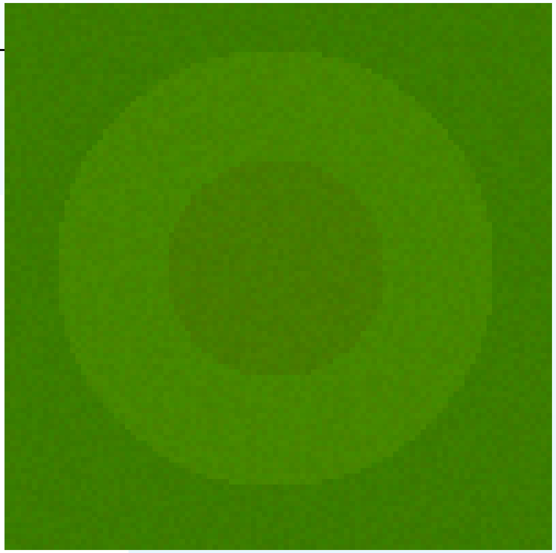
$$Y = D^{-1/2} X$$

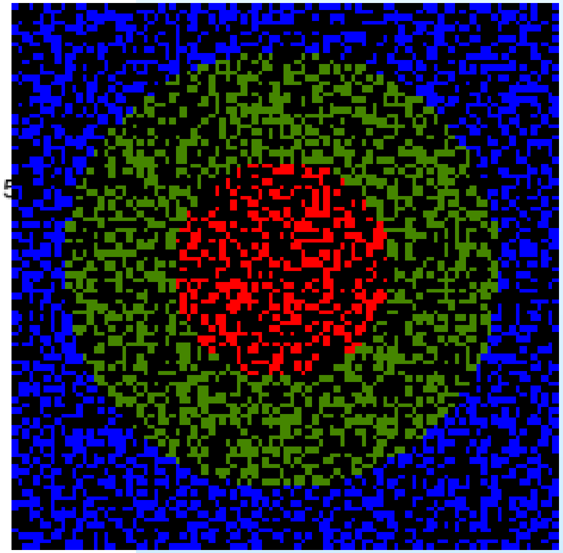
- In our algorithm, we adopt this weighting in order to avoid Ng's normalization.

Experimental results (1)

- Example







Prototype pixels

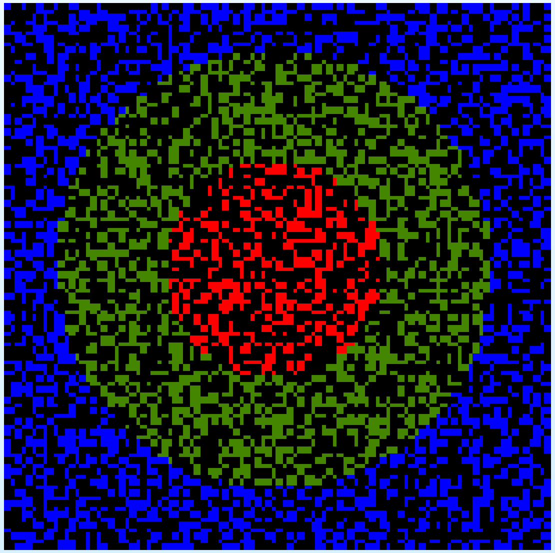
Final labelling (1)

- Assignation of unlabelled pixels
  - Credal filter [Vannoorenberghe 08]
    - Framework : belief functions theory
    - Measures
      - membership degree of a given pixel to each class
      - doubt degree of pixel assignment
    - Doubt degree of pixel assignment
      - increases / min. distance between its color and the class centers
      - decreases / number of its neighbors assigned to the same class

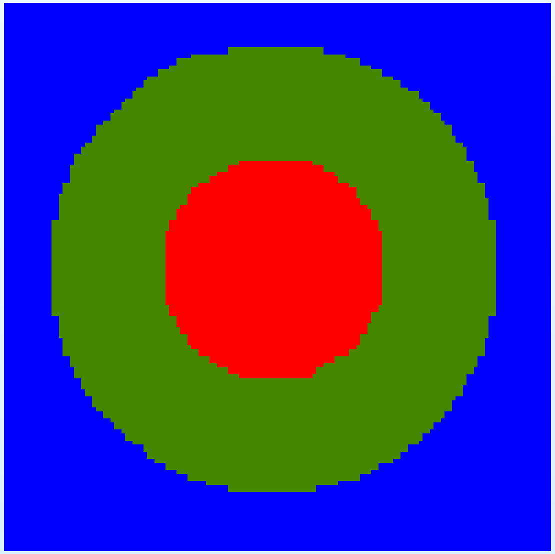
Final labelling (2)

- Assignation of unlabelled pixels
  - Result on the synthetic image

Original image



Prototype pixels

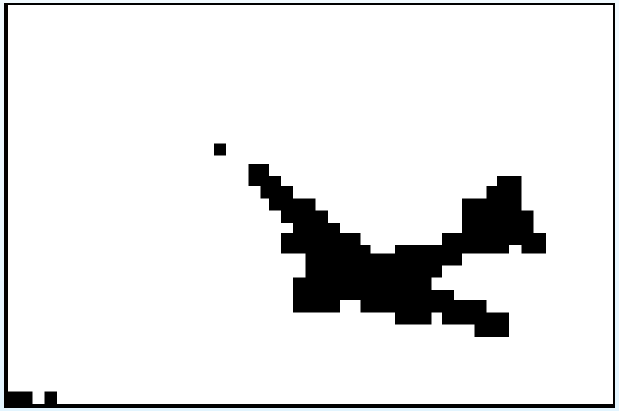


Final segmentation

Experimental results(2)

Natural image : '*Plane*' (Berkeley #3096)

Image



Robust path-based spectral clustering [Chang 2008]

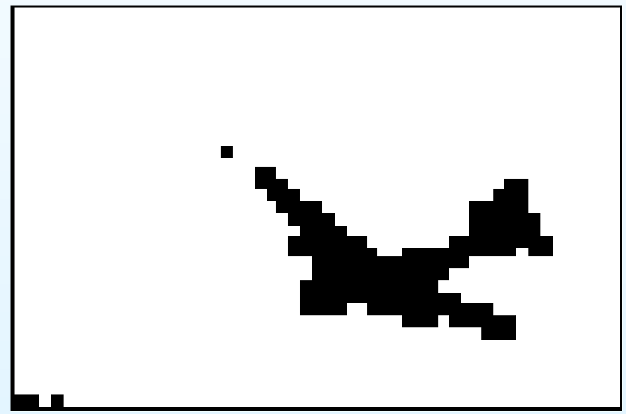
Spectral clustering

Spatial-color spectral clustering

Experimental results(2)

Natural image : '*Plane*' (Berkeley #3096)

Image



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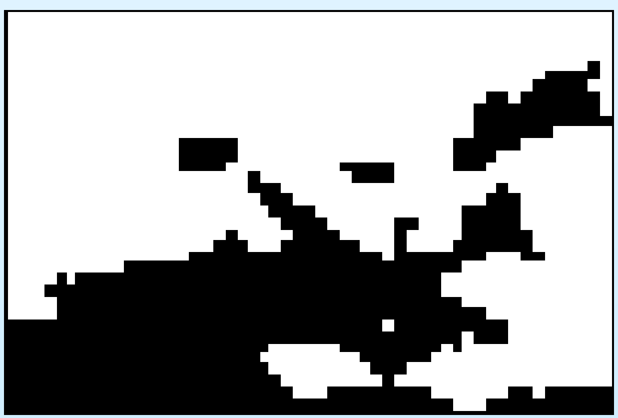
Spectral clustering

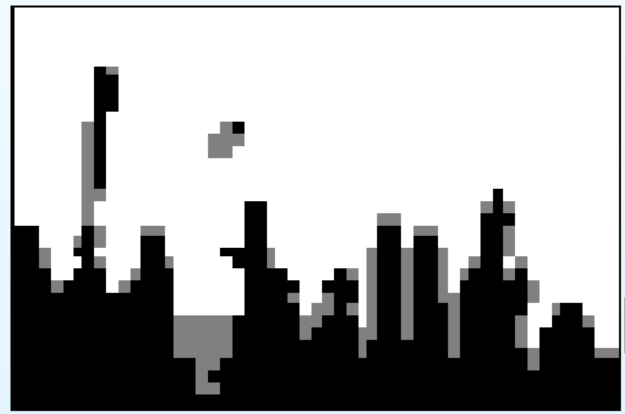


Spatial-color spectral clustering

Experimental results(2)

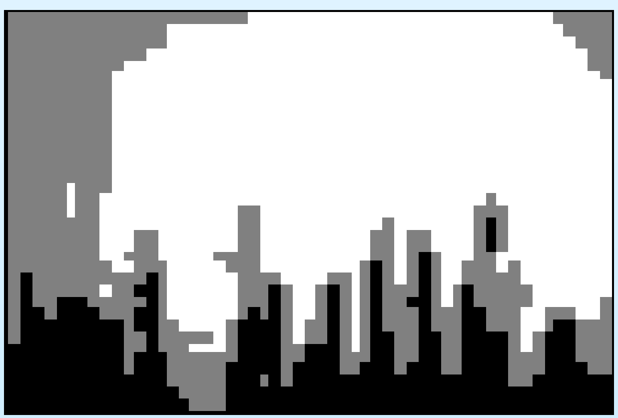
Natural image : '*Moon*' (Berkeley #238011)

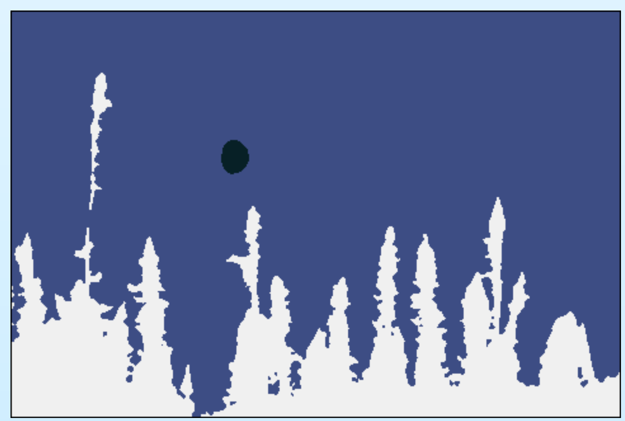
Image



Robust path-based spectral clustering [Chang 2008]

Spectral clustering





Spatial-color spectral clustering

Conclusions and future work

- Advantages of this approach
  - Ability to separate distributions with large color overlapping.
  - Similarity matrix between colors with homogeneity and connectedness.
  - Spectral clustering without gaussian filtering.
- Shortcomings and prospects
  - Influence of the color space.
  - Similarity function .
  - Adaptation of the spectral clustering.